

# SCIENTIFIC AMERICAN

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MATHEMATICAL GAMES

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Source: *Scientific American*, Vol. 209, No. 1 (July 1963), pp. 134-145

Published by: Scientific American, a division of Nature America, Inc.

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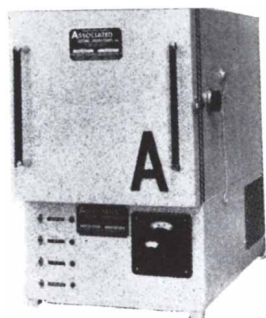


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# MATHEMATICAL GAMES

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*Topological diversions, including  
a bottle with no inside or outside*

by Martin Gardner

*Three jolly sailors from  
Blaydon-on-Tyne  
They went to sea in a bottle by Klein.  
Since the sea was entirely inside  
the hull  
The scenery seen was exceedingly dull.*

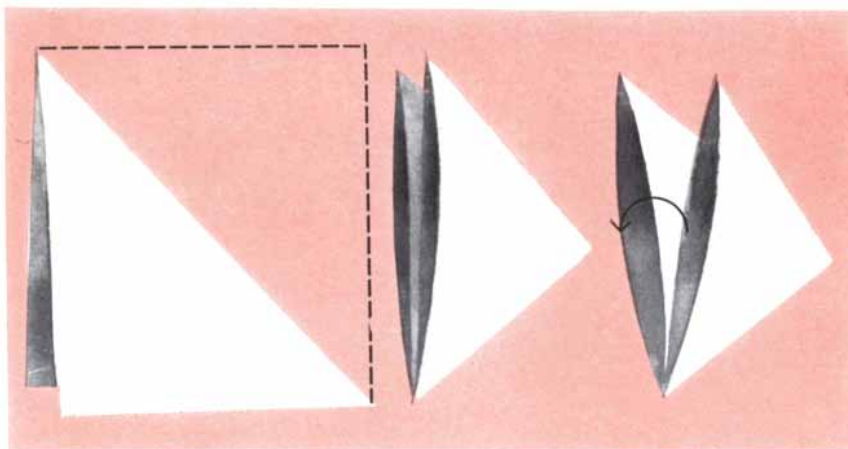
*The Space Child's Mother Goose,*  
by Frederick Winsor

To a topologist a square sheet of paper is a model of a two-sided surface with a single edge. Crumple it into a ball and it is still two-sided and one-edged. Imagine that the sheet is made of rubber. You can stretch it into a triangle or circle, into any shape you please, but you cannot change its two-sidedness and one-edgedness. They are topological properties of the surface, properties that remain the same regardless of how you bend, twist, stretch or compress the sheet.

Two other important topological invariants of a surface are its chromatic number and Betti number. The chromatic number is the maximum number of regions that can be drawn on the surface in such a way that each region has

a border in common with every other region. If each region is given a different color, each color will border on every other color. The chromatic number of the square sheet is 4. In other words, it is impossible to place five differently colored regions on the square so that each pair has a boundary in common. (This should not be confused with the famous four-color theorem, which involves maps with any finite number of regions.) The Betti number, named after Enrico Betti, a 19th-century Italian physicist, is the maximum number of cuts that can be made without dividing the surface into two separate pieces. If the surface has edges, each cut must be a "crosscut": one that goes from a point on an edge to another point on an edge. If the surface is closed (has no edges), each cut must be a "loop cut": a cut in the form of a simple closed curve. Clearly the Betti number of the square sheet is 0. A crosscut is certain to produce two disconnected pieces.

If we make a tube by joining one edge of the square to its opposite edge, we create a model of a surface topologically distinct from the square. The surface is still two-sided but now there are two separate edges, each a simple closed curve. The chromatic number remains 4 but the Betti number has changed to 1. A crosscut from one edge to the other,



*Möbius surface constructed with a square*

# IBM reports on the field of applications programming.

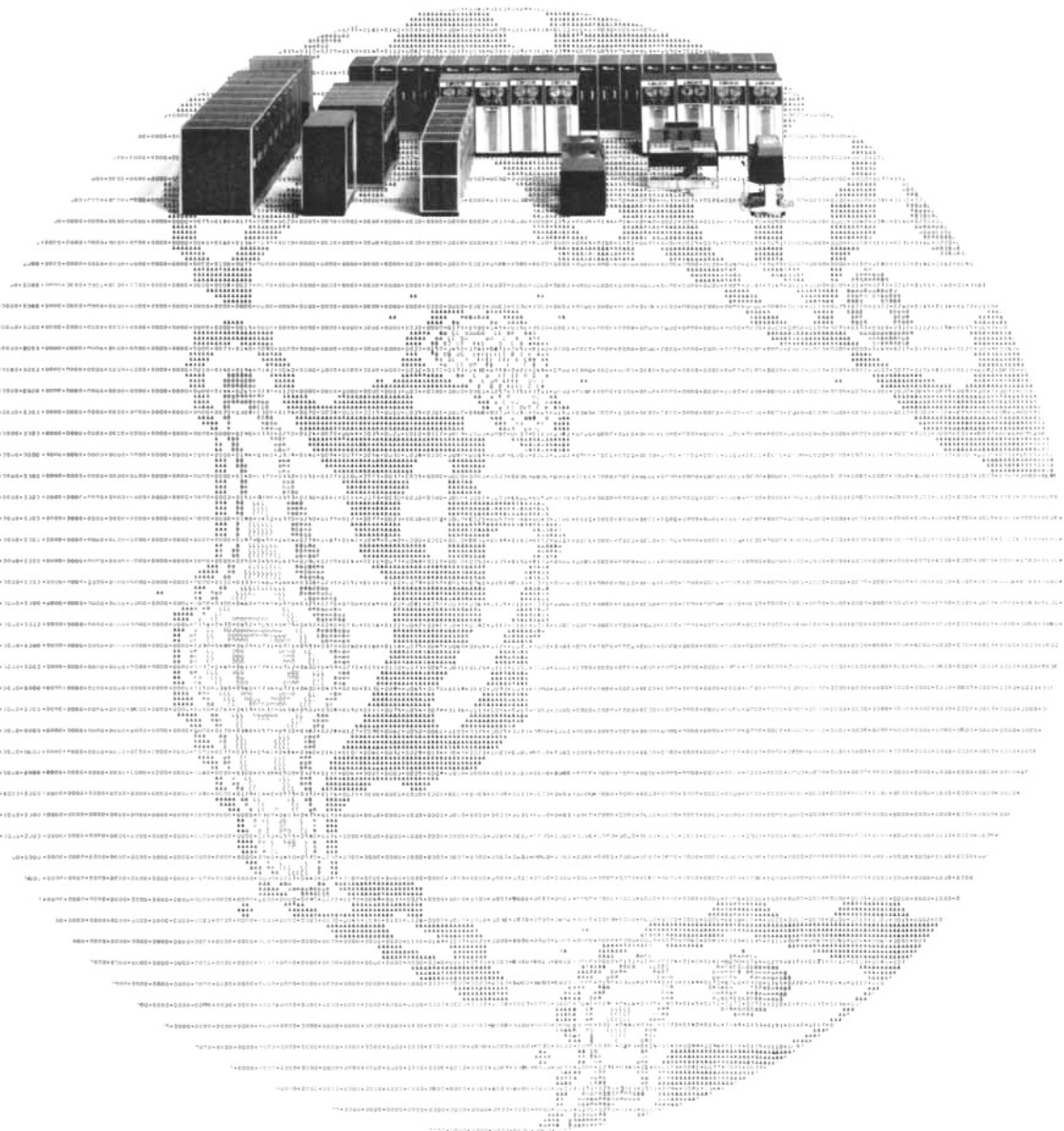
## Who trains computers for new jobs?

The program that a computer follows in doing its work is a logical series of simplified directions. To develop these, the programmer must thoroughly understand the problem he wishes the computer to solve. IBM has studied its customers' problems diligently and has worked out families of applications to which general program systems may be most efficiently applied.

In an unusual example of applications programming, IBM assisted the U. S. Weather Bureau in programming a system for global weather simulation on an IBM STRETCH (7030). The computer program is based upon a mathematical model formulated by the General Circulation Research Laboratory at the Weather Bureau, for research on the problems of long-range forecasting. In this massive system the basic processes of weather are simulated for the entire globe in a more detailed and

fundamental manner than ever before. The simulated weather is calculated for as many as 10,000 grid points at each of nine atmospheric levels and for time intervals as small as five minutes, so that over ten billion calculations may be required to simulate the weather for a single day. Even in the highly efficient STRETCH language, over 15,000 instructions were required for this versatile system, which incorporates such varied factors as radiation, turbulence, clouds, oceans, mountain ranges, and forests.

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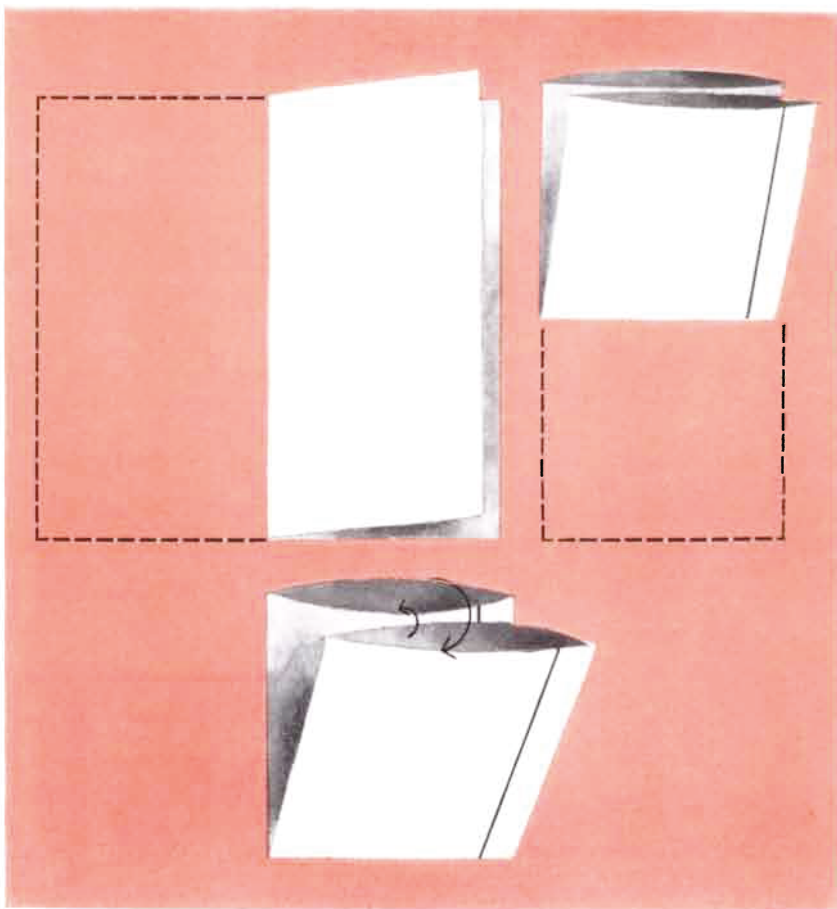
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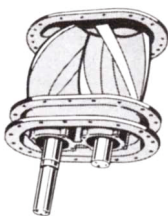


*Torus surface folded from a square*

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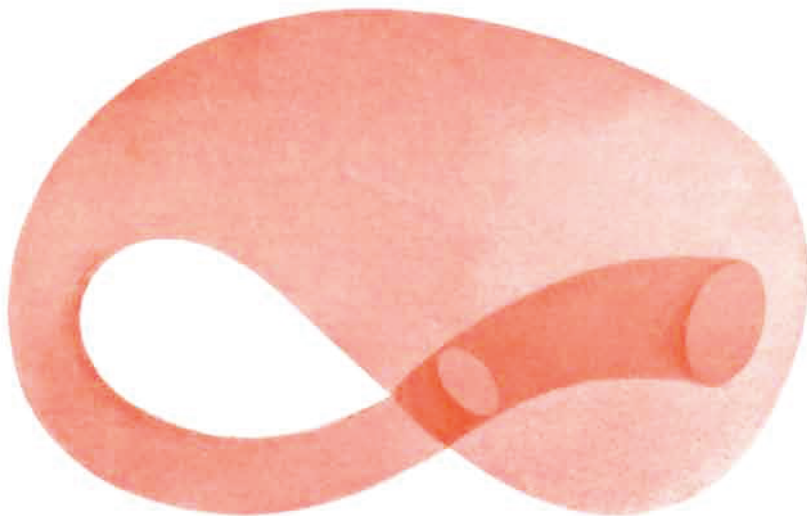
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although it eliminates the tube, allows the paper to remain in one piece.

A third type of surface, topologically the same as the surface of a sphere or cube, is made by folding the square in half along a diagonal and then joining the edges. The surface continues to be two-sided but all edges have been elimi-

nated. It is a closed surface. The chromatic number continues to be 4. The Betti number is back to 0: any loop cut obviously creates two pieces.

Things get more interesting when we join one edge of the square to its opposite edge but give the surface a half-twist before doing so. You might sup-



*Klein bottle: a closed surface with no inside or outside*



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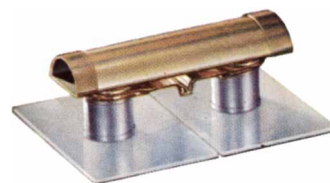
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## Decision-Making: Logistics Support... What, Where, When?

Centuries ago the critical factor in logistics support was providing basic supplies—food, armaments, raw materials for simple industries. A few decades ago, carrying capacity—sea and land transportation—ruled as the decisive element. Within the last decade, a new critical element in logistics support has emerged. It has been created by the complex, interfacing governmental, industrial and military structure of today. This new factor is up-to-the-minute information—gathered from afar, varied in content, immense in volume.

To help provide and control this flow of information, SDC scientists, and engineers

have helped create a new technology: information systems which aid managers in determining the “what, where and when” of logistics support for world-wide and continental activities and forces.

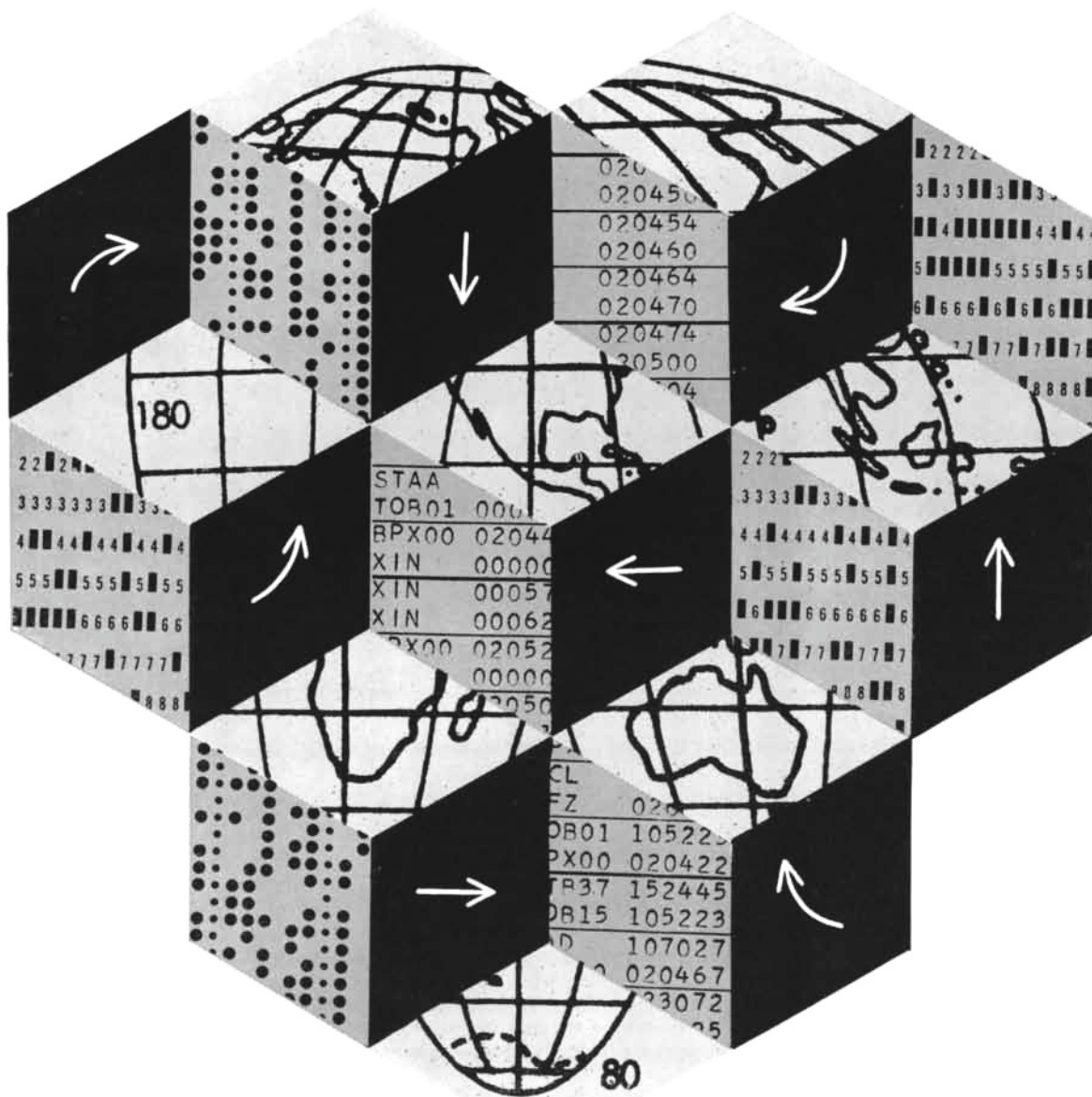
In developing these systems which provide information processing assistance, SDC scientists and engineers have evolved an interdisciplinary approach. Teams of operations research scientists, engineers, computer programmers and human factors scientists work together in these major system development steps: analyzing the system, synthesizing the system, instructing computers within the system, training

the system, evaluating the system—and helping adapt the system to the changing needs of its users.

Scientists and engineers interested in joining this growing field are invited to write Dr. H. H. Best, SDC, 2430 Colorado Ave., Santa Monica, California. Positions are open at SDC facilities in Santa Monica; Washington, D.C.; Lexington, Massachusetts; Paramus, New Jersey; Dayton, Ohio. “An equal opportunity employer”



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pose that this cannot be done with a square piece of paper, but it is easily managed by folding the square twice along its diagonals, as shown in the illustration on page 134. Tape together the pair of edges indicated by the arrow in the last drawing. The resulting surface is the familiar Möbius strip, first analyzed by A. F. Möbius, the 19th-century German astronomer who was one of the pioneers of topology. The model will not open out, so it is hard to see that it is a Möbius strip, but careful inspection will convince you that it is. The surface is one-sided and one-edged, with a Betti number of 1. Surprisingly, the chromatic number has jumped to 6. Six regions, of six different colors, can be placed on the surface so that each region has a border in common with each of the other five.

When both pairs of the square's opposite edges are joined, without twisting, the surface is called a torus. It is topologically equivalent to the surface of a doughnut or a cube with a hole bored through it. The top illustration on page 136 shows how a flat, square-shaped model of a torus is easily made by folding the square twice, taping the edges as shown by the solid gray line in the second drawing and the arrows in the last. The torus is two-sided, closed (no-edged) and has a chromatic number of 7 and a Betti number of 2. One way to make the two cuts is first to make a loop cut where you joined the last pair of edges (this reduces the torus to a tube) and then a crosscut where you joined the first pair. Both cuts, strictly speaking, are loop cuts when they are marked on the torus surface. It is only because you make one cut before the other that the second cut becomes a crosscut.

It is hard to anticipate what will happen when the torus model is cut in various ways. If the entire model is bisected by being cut in half either horizontally or vertically, along a center line parallel to a pair of edges, the torus surface receives two loop cuts. In both cases the resulting halves are tubes. If the model is bisected by being cut in half along either diagonal, each half proves to be a square. Can the reader find a way (before the answer appears in this department next month) to give the model two loop cuts that will produce two separate bands interlocked like two rings of a chain?

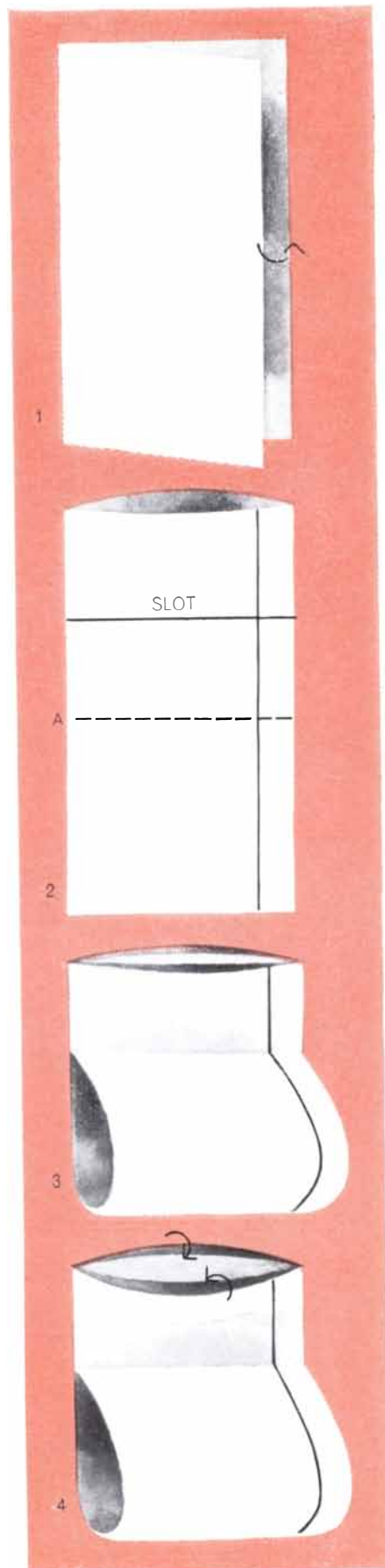
Many different surfaces are closed like the surface of a sphere and a torus, yet one-sided like a Möbius strip. The easiest one to visualize is a surface known as the Klein bottle, discovered in

1882 by Felix Klein, the great German mathematician. An ordinary bottle has an outside and inside in the sense that if a fly were to walk from one side to the other, it would have to cross the edge that forms the mouth of the bottle. The Klein bottle has no edges, no inside or outside. What seems to be its inside is continuous with its outside, like the two apparent "sides" of a Möbius surface.

Unfortunately it is not possible to construct a Klein bottle in three-dimensional space without self-intersection of the surface. The bottom illustration on page 136 shows how the bottle is traditionally depicted. Imagine the lower end of a tube stretched out, bent up and plunged through the tube's side, then joined to the tube's upper mouth. In an actual model made, say, of glass there would be a hole where the tube intersects the side. You must disregard this defect and think of the hole as being covered by a continuation of the bottle's surface. There is no hole, only an intersection of surfaces. This self-intersection is necessary because the model is in three-space. If we conceive of the surface as being embedded in four-space, the self-intersection can be eliminated entirely. The Klein bottle is one-sided, no-edged and has a Betti number of 2 and a chromatic number of 6.

Daniel Pedoe, a mathematician at Purdue University, recently wrote *The Gentle Art of Mathematics*. It is a delightful book, but on page 84 Professor Pedoe slips into a careless bit of dogmatism. He describes the Klein bottle as a surface that is a challenge to the glass blower, but one "which cannot be made with paper." Now, it is true that at the time he wrote this apparently no one had tried to make a paper Klein bottle, but that was before Stephen Barr, a science-fiction writer and an amateur mathematician of Woodstock, N.Y., turned his attention to the problem. Barr quickly discovered dozens of ways to make paper Klein bottles. He is currently at work on a book of topological recreations, to be published later this year by the Thomas Y. Crowell Company. It will be crammed with all sorts of elegant new topological models folded out of paper. Here I will describe only one of Barr's Klein bottles; one that enables us to continue working with a square and at the same time follows closely the traditional glass model.

The steps are given in the illustration at the right. First, make a tube by folding the square in half and joining the right edges with a strip of tape as



Folding a Klein bottle from a square

## WHERE IN THE WORLD

# ARE THE NAVAL LABORATORIES IN CALIFORNIA?

Physically, the U. S. Naval Laboratories in California are headquartered in California, which is no surprise. However, this is only part of the story . . . for, in a sense, their activities extend throughout the world. For instance, scientists and engineers from the laboratories work in such far-flung and divergent areas as Eniwetok, the Union of South Africa, Alaska, Australia and Hawaii, to name a few.

Laboratory personnel and facilities also occupy missile/satellite tracking ships in the Indian Ocean, participate in Operation Deep Freeze in the Antarctic, operate radar sites in Japan and probe the deep water secrets of, for example, the Mariannas Trench in the Philippines.

The extensive global activities of the Naval Laboratories in California match the collective and individual capabilities of each facility. Typical activities could involve, for example, oceanography studies (NEL); launching and tracking a Discoverer type satellite (PMR); arctic snow/ice research (NCEL); fleet ordnance development (NOTS); missile firing evaluation (NOLC); and nuclear radiation studies (NRDL).

Whether your scientific/technical interests are stimulated by working in a mainland laboratory or participating in a specialized research project in the distant tropics or the arctic, the U. S. Naval Laboratories in California offer a maximum challenge. Qualified scientists, engineers, and technicians who wish to explore employment opportunities at any of the laboratories listed below are invited to contact the Personnel Director, Dept. A, at the facility of their choice.

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**U. S. Naval Civil Engineering Laboratory (NCEL), Port Hueneme**

**U. S. Naval Ordnance Test Station (NOTS), China Lake and Pasadena**

**U. S. Naval Ordnance Laboratory (NOLC), Corona**

**U. S. Naval Radiological Defense Laboratory (NRDL), San Francisco**

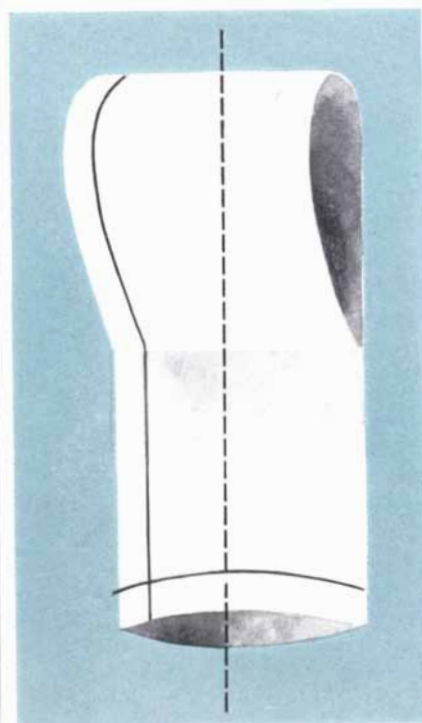
**U. S. Naval Personnel Research Activity (NPRA), San Diego**

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## U. S. NAVAL LABORATORIES IN CALIFORNIA

shown in Step 1. Cut a slot about a quarter of the distance from the top of the tube (Step 2), cutting only through the thickness of paper nearest you. This corresponds to the "hole" in the glass model. Fold the model in half along the broken line A. Push the lower end of the tube up through the slot (Step 3) and join the edges all the way around the top of the model (Step 4) as indicated by the arrows. It is not difficult to see that this flat, square model is topologically identical with the glass bottle shown in the bottom illustration on page 136. In one way it is superior: there is no actual hole. True, you have a slot where the surface self-intersects, but it is easy to imagine that the edges of the slot are joined so that the surface is everywhere edgeless and continuous.

Moreover, it is easy to cut this paper model and demonstrate many of the bottle's astonishing properties. Its Betti number of 2 is demonstrated by cutting the two loops formed by the two pairs of taped edges. If you cut the bottle in half vertically, you get two Möbius bands, one a mirror image of the other. This is best demonstrated by making a tall, thin model [see illustration below] from a tall, thin rectangle instead of a square. When you slice it in half along the broken line (actually this is one long loop cut all the way around the surface), you will find that each half opens out into a Möbius strip. Both strips are par-



*Bisected bottle makes two Möbius strips*

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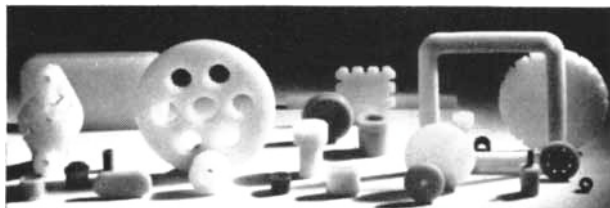
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# CORNING

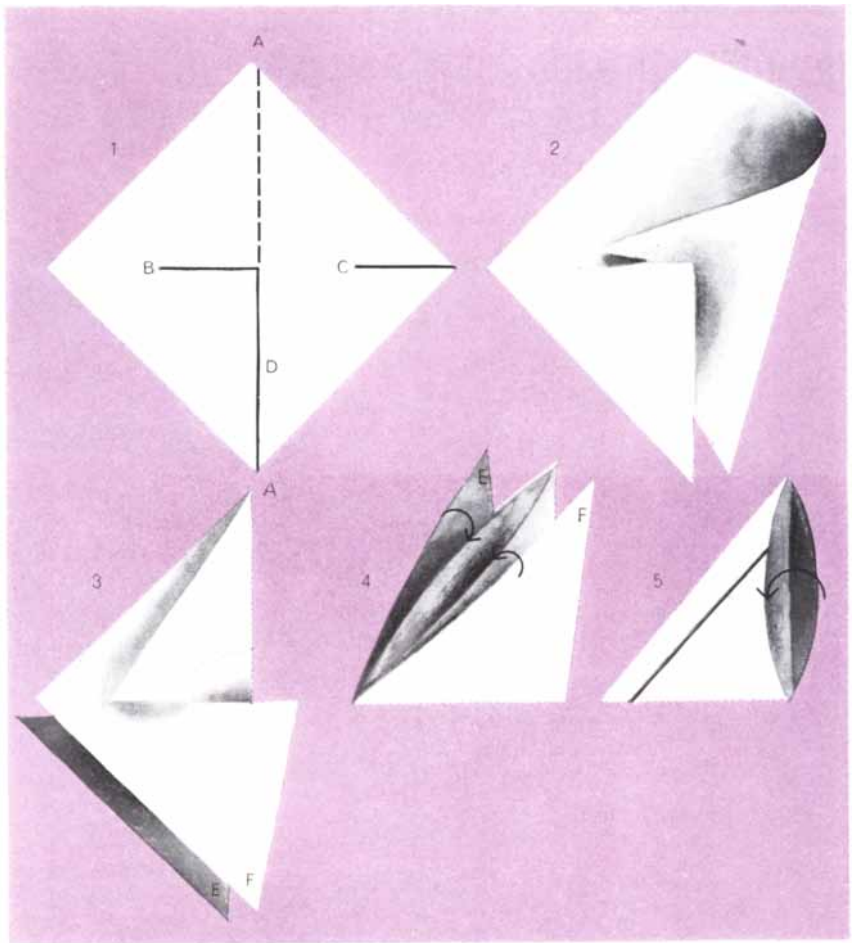
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*Folding a cross-cap and projective plane from a square*

tially self-intersecting, but you can slide each strip out of its half-slot and close the slot, which is not supposed to be there anyway.

If the bottle can be cut into a pair of Möbius strips, of course the reverse procedure is possible, as described in the following anonymous limerick:

*A mathematician named Klein  
Thought the Möbius band was divine.*

*Said he: "If you glue  
The edges of two,  
You'll get a weird bottle like mine."*

Surprisingly, it is possible to make a single loop cut on a Klein bottle and produce not two Möbius strips but only one. A great merit of Barr's paper models is that problems like this can be tackled empirically. Can the reader discover how the cut is made?

The Klein bottle is not the only simple surface that is one-sided and no-edged. A surface called the projective plane (because of its topological equivalence to a plane studied in projective geometry) is similar to the Klein bottle in both

respects as well as in having a chromatic number of 6. As in the case of the Klein bottle, a model cannot be made in three-space without self-intersection. A simple Barr method for folding such a model from a square is shown in the illustration above. First cut the square along the solid black lines shown in Step 1. Fold the square along the diagonal A-A', inserting slot C into slot B as shown in Steps 2 and 3. You must think of the line where the slots interlock as an abstract line of self-intersection. Fold up the two bottom triangular flaps E and F, one on each side (Step 4), and tape the edges as indicated.

The model is now what topologists call a cross-cap, a self-intersecting Möbius strip with an edge that can be stretched into a circle without further self-intersection. This edge is provided by the edges of cut D, originally made along the square's diagonal. Note that unlike the usual model of a Möbius strip, this one is symmetrical: neither right- nor left-handed. When the edge of the cross-cap is closed by taping it as shown in Step 5, the model becomes a projec-

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For name of nearest ACCUTRON dealer and free booklet, write Bulova Watch Co., Inc., Dept. SA, 630 Fifth Avenue, New York 20, New York. Don't you owe it to yourself to wear ACCUTRON instead of a watch?

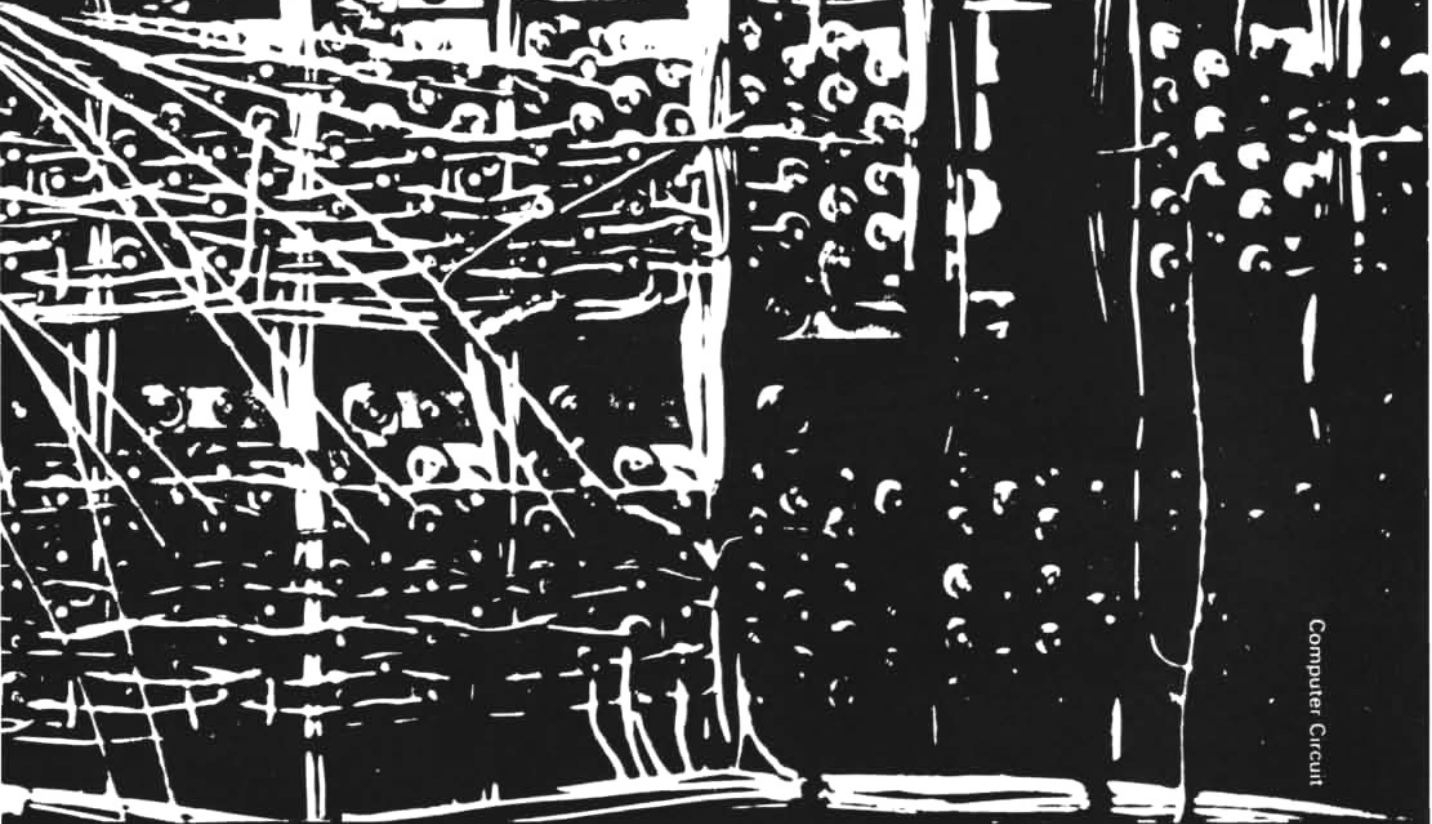
Read the ACCUTRON guarantee of accuracy! ACCUTRON is guaranteed by Bulova not to gain or lose more than one minute a month in actual daily use on your wrist. For one full year from date of purchase, the authorized jeweler from whom you purchased your ACCUTRON timepiece will adjust it to this tolerance, if necessary, without charge.



ACCUTRON "214" Brilliant stainless steel case, raised dial markers. \$125\*

## ACCUTRON A RESEARCH BREAKTHROUGH BY **BULOVA**

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Computer Circuit



Lincoln Laboratory, a research center of the Massachusetts Institute of Technology, conducts investigations in advanced electronics directed toward the solution of problems of national defense and space exploration. The *General Research* program provides a background of experience and ideas for programs concerned with specific defense and space problems, as well as a continuing source of contributions to electronics science and technology. All qualified applicants will receive consideration for employment without regard to race, creed, color or national origin. Lincoln Laboratory, Massachusetts Institute of Technology, Box 18, Lexington 73, Massachusetts.

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tive plane. You might expect it to have a Betti number of 2, like the Klein bottle, but it does not. It has a Betti number of 1. No matter how you loop-cut it, the cut produces either two pieces or a piece topologically equivalent to a square sheet that cannot be cut again without making two pieces. If you remove a disk from anywhere on the surface of the projective plane, the model reverts to a cross-cap.

The chart at the right summarizes all that has been said. The square diagrams in the first column show how the edges join in each model. Sides of the same color join each to each, with the direction of their arrows coinciding. Corners labeled with the same letter are corners that come together. Broken lines are sides that remain edges in the finished model. Next to the chromatic number of each model is shown one way in which the surface can be mapped to accommodate the maximum number of colors. It is instructive to color each sheet as shown, coloring the regions on both sides of the paper (as though the paper were cloth through which the colors soaked), because you must think of the sheet as having zero thickness. An inspection of the final model will show that each region does indeed border on every other one.

Last month's problem was to determine the length of the red stripe on a barber's pole, given that the pole is four feet high and that the stripe cuts the elements of the pole (vertical lines parallel with the axis) at a constant angle of 60 degrees.

If a right triangle is wrapped around any type of cylinder, the base of the triangle going around the base of the cylinder, the triangle's hypotenuse will trace a helix on the cylinder. Think of the red stripe of the barber's pole as the hypotenuse of a right triangle, then "unwrap" the triangle from the cylinder. The triangle will have angles of 30 and 60 degrees. The hypotenuse of such a triangle must be twice the altitude. (This is easily seen if you place two such triangles together to form an equilateral triangle.) In this case the altitude is four feet, so that the hypotenuse (red stripe) is eight feet.

The interesting part of this problem is that the length of the stripe is independent not only of the diameter of the cylinder but also of the shape of its cross section. The cross section can be an irregular closed curve of any shape whatever; the answer to the problem remains the same.

	SURFACE	CHROMATIC NUMBER	SIDES	EDGES	BETTI NUMBER
	<p>SQUARE (OR DISK)</p>	<p>4</p>	2	1	0
	<p>TUBE</p>	<p>4</p>	2	2	1
	<p>SPHERE</p>	<p>4</p>	2	0	0
	<p>MÖBIUS STRIP</p>	<p>6</p>	1	1	1
	<p>TORUS</p>	<p>7</p>	2	0	2
	<p>KLEIN BOTTLE</p>	<p>6</p>	1	0	2
	<p>PROJECTIVE PLANE</p>	<p>6</p>	1	0	1

Topological invariants of seven basic surfaces